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大正二年十一月三十日 November 30th, 1913.

# 帝國學士院第二部メモアル

# MEMOIRS OF THE IMPERIAL ACADEMY

## SECTION II.

## The Structure of Mercury Lines examined by an Echelon Grating and a Lummer-Gehrcke Plate.

BY

H. Nagaoka and T. Takamine.

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# The Structure of Mercury Lines examined by an Echelon Grating and a Lummer-Gehrcke Plate*

BY

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§ 1. *Introduction.*—The structure of mercury lines has been investigated by a number of physicists, following different methods and using different instruments. The results are, however, generally not in good agreement; this may be due, on the one hand, to the imperfection of the optical instruments as well as to the difference in the method of using them, and, on the other, to the non-identity of the source of light.

The simplest way of finding the position of the satellites would be to photograph the spectrum with a concave grating of high resolving power, but the method is at present open to objection, as the photographs may contain false lines arising from irregularities in the ruling. If it were possible to have a grating ruled with a sufficient number of lines, such that its resolving power is great enough to compete with that of interferometers, the problem would be easily solved. To obtain a resolving power in the spectrum of a third order equal to that of the Lummer-Gehrcke plate used by v. Baeyer, it would be necessary to rule a grating of about 40 cm. length with 568 lines to the millimetre, as in Rowland's gratings. The technical difficulty of the problem of constructing a perfect screw with a straight guide, and of finding a diamond point to rule a total length of several miles of lines without wear, is not yet overcome, and the grating used *singly* is not free from ghosts, unless some means of discriminating them from the true lines be devised. This is always essential with all kinds of instruments, but the question has not yet been solved with respect to the spectra obtained with a concave grating.†

With the echelon spectroscope, the presence of ghosts can hardly be avoided, and

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* Owing to great delay in the publication, a few passages have been added, especially with reference to the works which appeared after the paper was read.

† Wood, Phil. Mag. **25**, p. 443, 1913. The above remark applies to the structure of mercury lines as given by Wood in the paper cited. It is much to be regretted that the structure of the green line (5461), which is perhaps the best known, has not been given, for it would have been easy to compare the results obtained with different interferometers with that given by a *single* concave grating.

the order of spectrum, for the satellites at some distance from the principal line, gets extremely ambiguous from overlapping. Although the instrument suffers from these two defects, the intensity of light attained in it is far superior to that in other interferential apparatus, as it concentrates the light mostly on one or two orders of spectra. An interesting paper by Stansfield and Walmsley* sheds some light on the nature of the ghosts to be observed with an echelon grating, but the elimination of the ghosts can be easily effected by crossing the spectra according to the method of Gehrcke and von Baeyer,† which is analogous to that of crossed prisms, first used by Kundt in investigating the anomalous dispersion. The echelon may be crossed by a grating, or by another echelon, or by Fabry-Perot air plate, or by a Lummer-Gehrcke plate; we followed the last method to clear some doubts still attaching to the nature of the satellites; the reason will be explained later.

Fabry-Perot's air plate between half-silvered plane glass is nearly free from errors, which are almost without exception to be found in the echelon grating and the Lummer-Gehrcke plate, but it is extremely tedious to place the different statellites in order, especially when the line has a complicated structure. The spectra obtained by crossing the air plate with the echelon grating and the Lummer-Gehrcke plate are highly interesting, and an account of their study will be reserved for a future communication.

The intensity of the satellites has not yet been accurately measured, nor has the regularity in their position been well ascertained. According to our investigation, the intensity as well as the position of satellites are not altogether irregular; this fact will be of a special interest to those who wish to unravel the secrets of atomic structure.

§ 2. *Echelon Grating.* The echelon grating and the Lummer-Gehrcke plate used in the present investigation were made by Hilger. The former was of the following dimensions:—

| | |
|---|---|
| Thickness of the plate ... ... ... ... ... ... | 9.350 mm. |
| Number of plates ... ... ... ... ... ... ... | 35 |
| Steps ... ... ... ... ... ... ... ... ... | 1.0 mm. |
| Length ... ... ... ... ... ... ... ... ... | 32.73 cm. |

The thickness given by Hilger agrees with that obtained by an actual measurement made on a piece of a prism, cut out from the same plate as that used for constructing the echelon‡, by means of Abbe's contact micrometer (Dickenmesser), reading to one micron. It was compared with a nickel-steel etalon, previously standardized at the Bureau International des Poids et Mesures. The above prism (refracting angle 60° 0′ 00.9″) was used in finding the indices of refraction of the echelon plate for mercury lines, employing a spectrometer (diameter of the graduated circle 30 cm., reading to 1″ by microscopes placed diametrically opposite) for the purpose. The values obtained can be almost exactly expressed by means of either

* Stansfield and Walmsley, Phil. Mag. **23**, p. 25, 1912.

† Gehrcke and v. Baeyer, Ann. d. Phys. **20**, p. 267, 1906.

‡ Lunelund (Ann. d. Phys. 34, p. 505, 1911) has some doubt about the thickness of our echelon. The number quoted by Lunelund may refer to another echelon, of which there are several in Japan.

Cauchy's or Hartmann's formula. It is found on interpolation that the numbers given by Hilger for the refractive indices differ slightly in the fourth decimal.

The values of the constants in Cauchy's formula for the index of refraction, $\mu=A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}$, are found to be as follows:—

$$A=1.555055, \qquad B=5.9595\times10^5, \qquad C=1.9514\times10^{12}.$$

The separation of spectra of consecutive orders in the position of minimum deviation is given by the formula:

$$d\lambda_{max}=\frac{\lambda^2}{t\left\{(\mu-1)-\lambda\frac{d\mu}{d\lambda}\right\}}\cdot$$

The values of $d\lambda_{max}$ (in Å.U.) calculated from the above by using the constants in Cauchy's formula of dispersion and the indices of refraction are as follows:—

| $\lambda$ | 5790.5 | 5461.0 | 4916.4 | 4358.6 | 4078.1 | 4046.8 |
|---|---|---|---|---|---|---|
| $\delta$ | 43°51′50″ | 44°6′54″ | 44°39′25″ | 45°28′56″ | ......... | 46°8′23″ |
| $\mu$ (15° c) | 1.57456 | 1.57725 | 1.58305 | 1.59181 | 1.59790 | 1.59873 |
| $d\lambda_{max}$ | 0.5805 | 0.5090 | 0.4058 | 0.3002 | 0.2547 | 0.2497 |

The homogeneity of glass, as arranged in the echelon grating, was tested by placing the apparatus between two crossed Nicol prisms on a large Paalzow's polariscope made by Schmidt and Haensch. The dark field was then dimly lighted, but none of the polarisation bands was to be detected, showing that the pile of plates, though not entirely free from strain by one sided clamping, was almost homogeneous. That the echelon was slightly strained could at once be proved, since a beam of parallel rays, on passing through it, did not issue as such, but the pile acted somewhat like a lens. This fact is already well known, so that it will be only necessary to note it.

§ 3. *Lummer-Gehrcke Plate.* The Lummer plate was of the same kind of glass as that of the echelon grating. The plate was 20 cm. long, 3.5 cm. wide and 1.09116 cm. thick, at 18°. The last number was determined by using Abbe's contact micrometer. The indices of refraction were determined directly by means of Abbe's crystal refractometer, the constant of the instrument being checked by using a quartz plate, and also by the prism of the echelon plate, whose indices of refraction were already measured by another instrument of high accuracy. The indices thus found were less by 7 or 8 in the 4th decimal place than those of the echelon plate; they were conformable to Cauchy's as well as to Hartmann's formula. Plotting the values of $\mu$ on Hartmann's dispersion net, the observed points were found to lie on a straight line.

The value of $d\lambda_{max}$ may be calculated in the following manner. The order of the spectrum $h$, is given by

$$h=\frac{2t\sqrt{\mu^2-\sin^2 i}}{\lambda}, \tag{1}$$

where $t$ is the thickness and $i$ the angle of exit.

Differentiating it with respect to $\lambda$,

$$h^2\lambda=4t^2\left(\mu\frac{d\mu}{d\lambda}-\frac{\sin 2i}{2}\frac{di}{d\lambda}\right). \tag{2}$$

Since $h$ is of the order $5\times10^4$ in the present experiment, we get the equation of finite difference:

$$h\lambda^2\Delta h=-2t^2\sin 2i.\ \Delta i. \tag{3}$$

Equation (2) gives the following approximate relation:—

$$2t^2\sin 2i.\ \Delta i=\left(h^2\lambda-4t^2\mu\frac{d\mu}{d\lambda}\right)\Delta\lambda. \tag{4}$$

Hence by putting $\Delta h=1$, we have $\Delta\lambda=d\lambda_{max}$, so that

$$d\lambda_{max}=-\frac{\lambda^2}{h\lambda-\frac{4t^2\mu}{h}\frac{d\mu}{d\lambda}}. \tag{5}$$

The importance of introducing the correction for dispersion was first recognised by Laue* and used by v. Baeyer.†

For the grazing exit,

$$h=\frac{2t\sqrt{\mu^2-1}}{\lambda}, \tag{1'}$$

and the corresponding values of $d\lambda_{max}$ (in Å.U.) are as follows:

| $\lambda$ | 5790 | 5769 | 5461 | 4359 | 4078 | 4047 |
|---|---|---|---|---|---|---|
| $h$ | 45820 | 46000 | 48710 | 61980 | 66720 | 67200 |
| $d\lambda_{max}$ | 0.12090 | 0.11996 | 0.10659 | 0.06466 | 0.05596 | 0.05450 |

When the angle of exit $i$, deviates from 90° by 2° or 3°, it is necessary to introduce a small correction to $d\lambda_{max}$, which holds for $i=90°$. If the angle of exit can not be measured directly, it is approximately calculated by using (3):

$$\sin 2i=-\frac{h_0\lambda^2}{2t^2}\frac{\Delta h}{\Delta i}. \tag{3'}$$

* M. Laue, Ann. d. Phys. **13**, p. 163, 1904.

† v. Baeyer, Verh. d. Deutsch. Phys. Ges. **10**, p. 733, 1908.

It is generally sufficient to take $\Delta h=10$ or 15, find the corresponding $\Delta i$, and then calculate the correction. This is easily done on a photograph.

Recently, Koláček* raised the question whether the angle $i$ has the same meaning as that usually assumed in the calculation of $d\lambda$ without taking account of diffraction. According to our experiments, the image of the crossed spectra is not sharply defined as observed with a telescope adjusted for infinite distance. This is the point upon which the question was raised by Koláečk; it shows that the phenomenon observed with a plane parallel plate belongs to the problem of physical optics, and cannot properly be treated from the geometrical point of view. The deviation arising from this fact would be of little effect, as the values of $d\lambda$ are in most cases affected by only 1 or 2 figures in milli-Ångström units. This new departure ought to be put to a strict experimental test, when extreme accuracy in the measurement of $d\lambda$ is desirable.

In using the plate and the echelon grating, care was taken to protect them from changes of temperature by enclosing them in wooden boxes, which were lined thickly with cork plate. Nobody remained in the room while the photographic plate was being exposed.

The lamp used in the present experiments was mostly of Arons-Lummer type, run by direct current of 10 amperes at 30 volts, and cooled by water current. Sometimes a Heraeus quartz lamp was used, but generally the lines were more distinct with the Lummer lamp, especially when it was well cooled and placed under low voltage. The photographs as well as visual observations show that the lines are better defined when the line of sight is parallel to the arc than when it is transverse.

This is well exemplified in the satellite next to the principal line of 5461. (Fig. 17).

In the present investigation, we did not enter upon experiments on the changes in lines caused by introducing different gases into the lamp, as was recently done by Wendt.†

The plate was provided with a right-angled prism, whose section is an isosceles triangle, as in the original form used by Lummer and Gehrcke, but it was found more convenient to change it to another, whose section is a right-angled triangle with one angle of about 22°.5, so that the plate can be used *à vision directe.*

The resolving power of the plate and of the echelon grating was nearly the same; for wavelength 0.5 $\mu$, it was 435000 for the echelon grating and 400000 for the plate. To obtain the same resolving power with a Rowland grating (15000 lines to the inch), a surface covering the length of 25 cm. must be ruled.

§ 4. *Arrangement.* In order to eliminate the ghosts, which inevitably appear in an echelon grating, the spectrum was crossed by the Lummer plate after the method of Gehrcke and v. Baeyer. For this purpose, the plate was placed horizontally and the echelon grating in such a position that the spectrum lines were vertical. The arrangement is shown in Fig. 1.

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* F. Koláček, Ann. d. Phys. **39**, p. 1431, 1912.

† Wendt, Ann. d. Phys. **37**, p. 535, 1912.

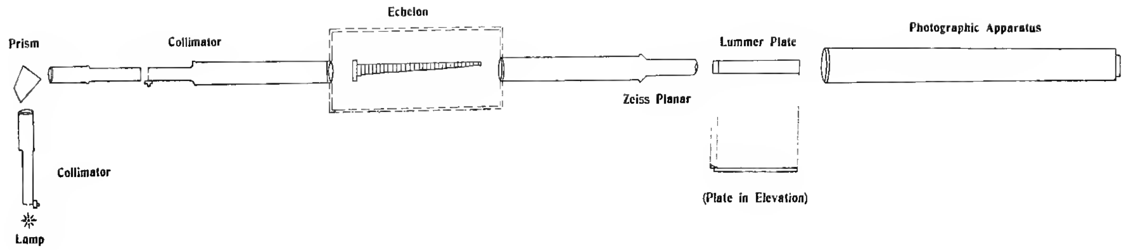


FIG. 1.

The light emerging from the echelon grating was made parallel by means of Zeiss planars (focal length: 7 cm., 5 cm., 3.5 cm.) and allowed to fall on the vertical face of the prism attached to the Lummer plate. The interference points were photographed by means of a Goerz anastigmat (focal length 21 cm., aperture 3 cm.), or by photographic lenses of 10 cm. aperture, having focal lengths of 70 cm. and 123 cm. respectively. The Goerz-lens was attached to a phototheodolite by Günther and Tegetmeyer, as there was an advantage in finding the angle of exit directly from the readings of the divided circle. In order to find the interference points of faint lines, it was necessary to use lenses of short focal length, but for the determination of the deviation of the satellites from the principal line, photographs obtained by lenses of longer focal lengths were used to secure more accuracy, without increasing the magnifying power of the micrometer, with which the relative position of the points was measured. For the latter purpose, the slit in front of the echelon grating was generally opened to a width of about 0.04 mm, so that the crossed images appeared on the photographic plate, as short lines instead of points. In this manner, the lines of the satellites could be placed between two spider lines of the micrometer, and their relative positions exactly determined. When the points were crowded, it was sometimes necessary to work with a slit only 0.01 mm. wide. In spite of the small quantity of light, which was admitted to pass through the echelon as well as the plate, the exposure needed did not exceed 5 or 6 hours even in the very insensitive part of the spectrum.

For the photography of yellow and green lines, the "panchromatic spectrum plate" of "Wratten and Wainwright" was used, and for the lines in the violet region the "Wratten process plate." Care was taken to develop and fix the plates under the same conditions.

The focussing of the lens was rather a tedious process, as it sometimes required more than a dozen photographs to obtain a sharp focus, especially for the lines in the almost invisible violet region. But in such region of the spectrum, exposure did not last more than 10 or 20 minutes, before obtaining a fairly good image. Thus we can utilise the bright image given by the echelon for shortening the time of exposure of photographic plates.

§ 5. *Crossed Spectra.* We tried different methods of crossing the spectra. The echelon spectrum was crossed with that obtained by a metallic plane grating ruled

on a Rowland engine, but owing to the faintness of light and the low resolving power of the grating, the results were by no means comparable with those obtained by the combination of the echelon and the Lummer plate. The same remark applies to the crossed spectra of two echelon gratings. The echelon above described was crossed with another of ten plates, having a resolving power of 140000*, made by Hilger and belonging to the Tokyo Higher Normal School. As it was necessary to work with very small slits, the quantity of light was generally insufficient to show the details of the spectra, though these were much superior to the crossed spectra of the echelon and an ordinary grating.

§ 6. *Advantage of Crossing the Spectra.* In all our observations, the angle of exit was nearly equal to 90°, so that by putting

$$i=90^\circ-a,$$

where $a$ is small, the equation giving the order of spectrum becomes

$$h^2\lambda^2=4t^2\{(\mu^2-1)+a^2\},$$

higher powers of $a$ being neglected.

Putting $\lambda=\lambda_0+\delta\lambda$, where $\lambda_0$ is the wavelength of the reference line, and $\delta\lambda$ the deviation of the wavelength of the satellites from it, we get

$$2h\lambda_0\delta\lambda=\{8t^2(\mu^2-1)-h^2\lambda^2_0\}+a^2.$$

With the crossed spectra given by the echelon grating and the Lummer plate, we may conveniently take for abscissae the distances of the satellites from the principal line on the echelon spectrum, which are therefore proportional to $\delta\lambda$, and regard the ordinates as proportional to the deviation $a$ from the grazing exit.

Consequently, the locus of interference points for the same value of $h$ and for different satellites must lie on a parabola given by the equation of the form:

$$y^2-a^2=bx,$$

where $a$, $b$ are constants depending on $h$. For consecutive values of $h$, the parabolas cut the $x$-axis at nearly equidistant intervals.

Thus, by tracing the curve, we are able to arrange the interference points according to the different values of $h$. This method is sometimes of great assistance in discriminating the positions of the satellites, especially when they are crowded together, as observed with the echelon or the plate only. In all of our measurements, we plotted the interference points from the readings of the micrometer with respect to $x$- and $y$- axis, in order to fix the order of the spectrum.

Fig.'s 8, 15, and 24 show at a glance the efficacy of the method, especially for 5790 and 4359, in which points belonging to different orders of the spectrum are mixed together.

* The instrument was placed at our disposal through the courtesy of Prof, Noda, to whom our best thanks are due,

In order to evaluate the relative positions of the satellites it is necessary to refer them to a line, which is sharply defined. Now, it is customary to refer them to the principal line, but as the latter is mostly broad and has hazy boundaries no accuracy can be expected to be obtained by taking it as the line of reference; indeed, the discrepancies existing between the results obtained by different observers sometimes seem to be attributable to the uncertainty in the position of the principal line. We have, therefore, in most cases used a well defined satellite as the reference line, and afterwards reduced it to the principal line.

With the plate, numerous orders of spectra can be measured by means of a micrometer, while with the echelon spectroscope only a single spectrum can be brought under test. Moreover, optical errors are simpler in the plate. Consequently, the result obtained with the latter is far more accurate than that obtained with the echelon. In all the calculations, which will be made hereafter, we take only the values obtained with the plate into account.

§ 7. *Interference Points.* Owing to the limit in the resolving power of the optical instruments, the fine structure of some of the lines can not be exactly ascertained. Although these can not be separated, we find that interference points arising from the combination of the echelon grating with the Lummer plate sometimes present a singular appearance. When the point is due to the intersection of simple strong lines, it has head and tail, if we may so call the tapered ends of the point which are in the direction of the tangent to the parabola joining the interference points of the same order. This is almost always shown in the photographs of the crossed spectra. With some points, however, the appearance is greatly modified, the head and tail, instead of being simple, presenting a complicated appearance. This is observed also when there is a weak satellite near a strong line, especially if the photograph is of a long exposure. Based upon this fact we may perhaps be able to draw, from the appearance of the points, some inference as to whether the line is simple or not, although the lines may not be distinctly separated. The same remark applies to the examination of the blackened line of photographic plates produced by a thermopile. (§ 8.)

When there is a ghost in the neighbourhood of a strong line, the interference points taper towards the ghost in the direction of the X-axis if it is due to the echelon, and in the Y-direction if it is attributable to the plate. An example of the former case is seen in the crossed spectra of 4078 (Fig 29.); only faint traces of the ghosts due to the plate were noticed in the line 5461. The plate was therefore nearly free from ghosts. A number of ghosts were found in the echelon spectrum of the lines 5461, 4359 and 4047, which may be seen in the photographs given in Fig.'s 16, 27, and 33.

§ 8. *Relative Intensity.* In spite of the numerous investigations on the positions of the satellites of mercury lines, very little has been done to measure their relative intensities.

The photometry of these lines may be made either by visual observations or by

mechanical registration. Each method has its merits and its demerits, but as the sensation of the eye is liable to various sources of error, which can not generally be controlled, we have chosen the latter means and carried out a photographic photometry by means of a linear thermopile.

We owe much to the careful studies of Hurter and Driffield* and those of Schwarzschild.† Hurter and Driffield say in their paper:

"In a theoretically perfect negative, the amount of silver deposited in the various parts of the plate is proportional to the logarithms of the intensities of light proceeding from the corresponding parts of the object." They found that the above statement is true within certain limits of the blackening of the photographic plate, which they call the "normal region" of the blackening curves.

Let $i_a$, $i_b$ denote the intensities of the satellites $a$ and $b$ respectively, and suppose we have a photograph taken in the normal region for both these satellites, then the corresponding blackenings $S_a$ and $S_b$ are given by

$$S_a=\log \varkappa\, i_a^m\, t^n,$$
$$S_b=\log \varkappa' i_b^{m'}\, t^{n'},$$

where $t$ is the time of exposure, and $\varkappa$, $m$, $n$, $\varkappa'$, $m'$, $n'$ are constants.

There has been much discussion on the nature of these constants‡ considered as functions of wave-lengths. But in the present case, the deviations of the satellites are only a fraction of the Ångström unit, so that we may treat the group as monochromatic. Moreover, the differences between the intensities do not seem to exceed the ratio 10: 1.

Hence we may assume

$$\varkappa=\varkappa', \quad m=m' \quad \text{and } n=n',$$

and obtain

$$\frac{S_a}{S_b}=\log\frac{i_a}{i_b}\,. \tag{1}$$

In the case of photographic photometry, let $J_1$ be the intensity of the incident light and $J_2$ that of the transmitted light; then we have for the blackening $S$,

$$S=\log\frac{J_1}{J_2}\,. \tag{2}$$

In the present experiments, the transmitted light falls on a thermopile and causes the galvanometer deflection $D$, which we may assume to be proportional to $J_2$, so that we have from (1) and (2)

$$\frac{S_a}{S_b}=\log\frac{i_a}{i_b}=\log\frac{J_{2.b}}{J_{2.a}}=\log\frac{D_b}{D_a}\,.$$

* Hurter and Driffield, Journ. of Chem. Indust **9**, p. 455, 1890.

† Schwarzschild, Publik. d. Kuffner. Sternwarte **5**, 1900.

‡ Stark, Ann. d. Phys. **35**, p. 461, 1911; Leimbach, Ann. d. Phys. **36**, p. 198, 1911.

Hence we see that the galvanometer deflections may be taken as the measure of the relative intensities, provided $S_a$ and $S_b$ lie within the normal region.

(*a*) *Photography.* The mercury lamp used in the present experiment was mostly of Arons-Lummer type, run by a direct current of 10 amperes at 30 volts, and cooled by a current of water.

Sometimes the Heräus quartz lamp was used, but generally the lines were more distinct with the Lummer lamp, especially when it was well cooled and run by weak current.

The echelon spectra of two successive orders were generally photographed in the position of the minimum deviation, by which the satellites were all seen between the two images of the principal line, which were nearly equal in their intensity. The space between the consecutive orders of the spectrum, which was equivalent to 0.509 Å. U. for the green line, ranged from 3 mm. to 9 mm. on the photographic plates.

The photographic plate we used was the "panchromatic spectrum plate" of "Wratten and Wainwright." The ordinary quarter-plate was cut into small squares, having 1.8 cm. side. To avoid the irregularity in the sensibility of the film at the edge of the plate, the portions 1.5 cm. from the sides were not used. For the developer, we used ferrous oxalate especially recommended by Hurter and Driffield for the density measurements. Care was taken to develop and fix the plates under the same conditions.

For the purpose of the photometry, it was necessary to photograph the echelon spectrum under several different degrees of exposure. Firstly, normal exposure for the principal line and the strong satellites, when the faint lines are mostly in the condition of under-exposure; secondly, normal exposure for the satellites of strong and mean intensity, when the principal line is overexposed; thirdly, normal exposure for the fainter lines, and so on. By the combination of the plates thus obtained and by their comparisons, we may arrive at an approximate value of the relative intensities.

(*b*) *Photometry.* The arrangement used for photometry is shown in Fig. 2.

The ray from a Nernst lamp N (110 volt, 1 amp.), placed in a large wooden box, was made parallel by a quartz lens L, and allowed to pass through the photographic plate P under examination, at normal incidence. The current feeding the lamp was carefully kept constant during the observation.

The photographic plate was attached to the end of a short tube to which delicate displacements could be given by the micrometer M, and the film surface of the plate was placed almost in contact with the slit S (17 mm. long, 0.05 mm. wide), so that the image of the spectrum lines was parallel to the slit.

A thermopile T after Rubens, consisting of 24 junctions of iron and constantan wires, was placed behind the slit, and the light passing through the slit fell on the junctions, which were linearly arranged.

The deflection of a D'Arsonval galvanometer G, connected with the pile, was read when successive displacements of 0.05 mm. were given to the plate. The

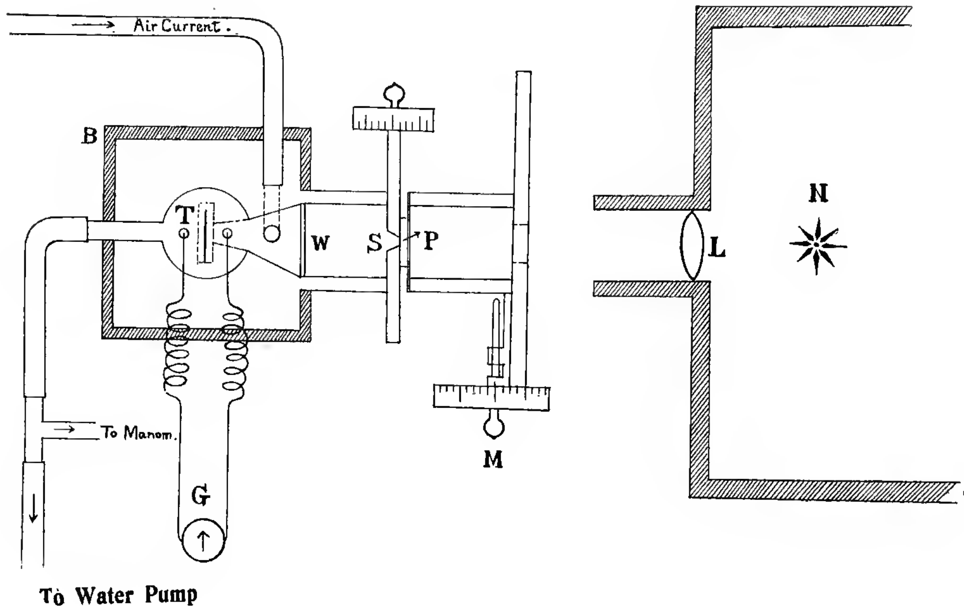


FIG. 2.

sensibility of the galvanometer was $5 \times 10^{-9}$ amp. per mm. deflection, at 1.5 m. distance.

The inertia of the instruments caused much trouble. After each displacement we had to wait some 10 seconds before the stationary state at the new position was reached, so that there was a great difficulty in keeping the zero of the galvanometer unchanged, since the metal case containing the thermopile got gradually heated, causing the temperature of the cold junction to rise slowly.

To avoid this difficulty, the metal case of the pile was closed on the front by a window W, of thin cover glass of microscope, and was placed within an wooden box filled with cotton. A slow current of dry and dustfree air, whose temperature was kept constant by passing it through a long metal worm immersed in a large water tank, was maintained through the metal case by a water pump, the pressure difference being 3 mm. of water.

By this means, the zero of the galvanometer remained nearly constant. Of course, it was necessary to pass the air current for several hours before starting the observation, and it took 5 or 6 hours to finish the examination of a plate.

From the 60 photographs taken of the green line ($\lambda$: 5461 Å. U.), 6 plates were chosen and examined in the manner above described.

After our paper was read, we received Koch's* paper on a registering microphotometer, in which a photoelectric cell is used in recording the intensity automatically. It has a decided advantage over the instruments above described in having very little inertia and in being independent of the fluctuation of the source of light, but the precautions as regards the development and examination of the photographic plate will not differ much from those above mentioned.

* P. P. Koch, Ann. d. Phys, **39**, p. 705, 1912.

The curves obtained by plotting the galvanometer deflections after each displacement were 15 in number. In most cases, 2 curves were drawn for the same plate which was displaced in reverse directions.

The accompanying figures (Fig. 3a, 3b, 3c) are the curves obtained from 3 different plates, showing three different stages of the blackening.

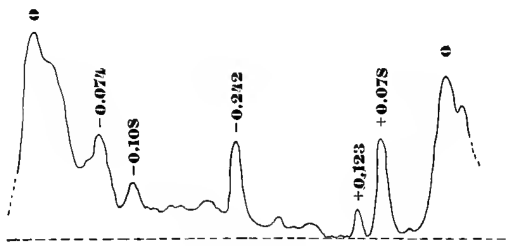


FIG. 3a.

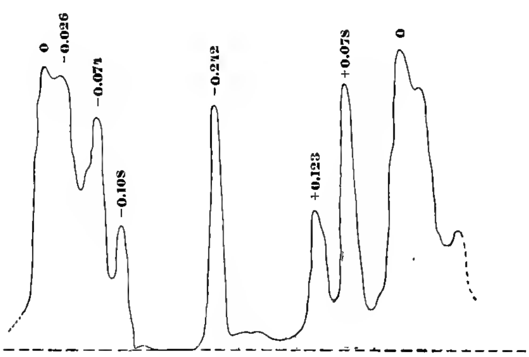


FIG. 3b.

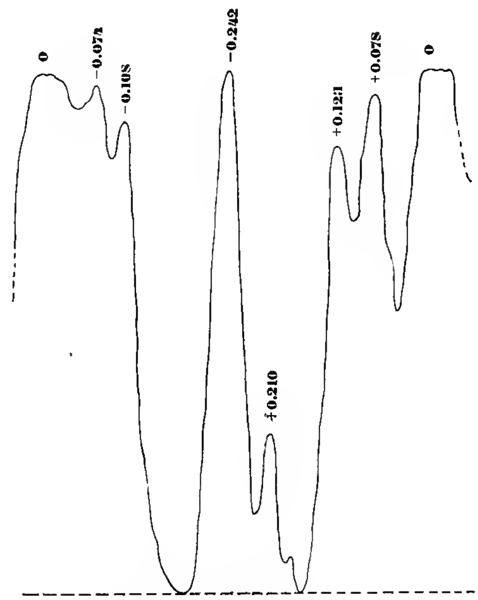


FIG. 3c.

In these figures, we have taken $\delta\lambda$ as abscissae and the galvanometer deflections as ordinates, but the latter in reverse direction, so that the heads of the curve represent the positions of the satellites. Zero lines are drawn to touch the lowest points of the curve, *i. e.* the points of largest deflections.

The slight inequality of the two principal lines seen in Fig. 3a is due to the setting of the echelon. The same inequality exists also in the cases of Fig. 3b and 3c, but is rendered indetectable by "over-blackening."

When examined with the echelon spectroscope alone, there appear a number of ghosts which, although very faint, are still quite misleading. This ambiguity was avoided in our experiments of the crossed spectra; we found that there are 8 satellites for the green line of mercury. These may be roughly classified into the following three groups according to their intensities.

(The numbers indicating the lines are $\delta\lambda$ expressed in Ångstrom unit).

(*A*) Strong lines

−0.242 −0.074 −0.026 +0.078

(*B*) Lines of middle intensity

−0.108 +0.123

(*C*) Faint lines

−0.054 +0.210

In Table I, our results on the relative intensities of the satellites are compared with those of other observers.

Table I.

| $\delta\lambda$ | Janicki* | Lunelund† | Nagaoka and Takamine |
|---|---|---|---|
| —0.242 | $\frac{1}{6}$ | 5 | 5.2 |
| —0.108 | $\frac{1}{10}$ | 3 | 2.5 |
| —0.074 | $\frac{1}{7}$ | 4 | 5.0 |
| —0.054 | | 2 | (1.5) |
| —0.026 | | 8 | 8.3 |
| —0. | 1 | 10 | 10.0 |
| +0.078 | $\frac{1}{3}$ | 6 | 5.8 |
| +0.123 | $\frac{1}{8}$ | 4 | 3.5 |
| +0.210 | | 2 | 1.3 |

($\delta\lambda$'s differ slightly according to authors, but their general agreement is hardly doubtful.)

Sometimes the relative intensities are expressed by the order of brightness. Table II shows this for different authors:

Table II.

| $\delta\lambda$ | Gale and Lemon‡ | v. Baeyer§ | Nagaoka and Takamine |
|---|---|---|---|
| —0.242 | 1 | 2 | 3 |
| —0.108 | 3 | 4 | 6 |
| —0.074 | 3 | 3 | 4 |
| —0.054 | | 5 | 7 |
| —0.026 | | 1 | 1 |
| +0.078 | 2 | 1 | 2 |
| +0.123 | 3 | 3 | 5 |
| +0.210 | 4 | 6 | 8 |

The relative intensity of the satellite -0,054 could not be measured directly from the curves, since this satellite lies between two strong lines which are very near each other, so that its image was much disturbed by the diffusion of the photo-

* Janicki, Ann. d. Phys. **19**, p. 36, 1906.
† Lunelund, Ann. d. Phys. **34**, p. 505, 1911.
‡ Gale and Lemon, Physik. Zeitschr. **11**, p. 209, 1910.
§ v. Baeyer, Verh. d. Deutsch. Physik. Ges. **9**, p. 84, 1907.
Ibid **10**, p. 733, 1908.

chemical action. The approximate value 1.5 for this line in Tab. I is an estimation obtained from the photograph of the crossed spectra.

As the photographs measured were taken with an echelon grating alone, there appeared a number of ghosts when the exposures were long. The intensities of these ghosts were in most cases very small, but they caused slight protuberances in the curves.

That these protuberances were not due to accidental causes, such as those arising from the change of intensity of the lamp, or that of the velocity of the air current etc, has been verified by the fact that on moving the micrometer backwards, exactly the same protuberances were noticed in the reverse direction.

The form of the curve representing the relative intensities of the satellites in their actual proportion would be as shown in the following figure (Fig. 4).

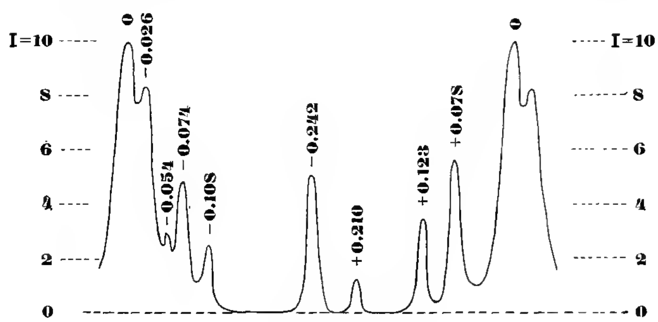

Fig. 4.

It would be superfluous to remark that Fig. 4 is not a direct copy obtained from a single photograph, but it only shows the result of a reduction from different curves.

A much more rigorous way would be to determine the normal region for each satellite, but in our case the fluctuation of light in the Arons-Lummer lamp was rather too large for such an accurate measurement.

The quartz mercury lamp of Heräus (110 volt, 3.5 amp.) emitted much steadier light, and several photographs of the green line were examined in the same manner. Although the data are too few to be described in detail, the distribution of intensities did not seem to be very different from that observed with the Arons-Lummer lamp.

It seems, however, that there is generally much difference in the distribution of intensities among these satellites when the source of mercury light is changed. We may refer to the photograph obtained by Gale and Lemon,* which shows the difference very markedly.

Even with the same lamp the relative intensity may be affected by the change of voltage, current, or pressure in the tube, etc. Of the many investigations, we may note that of Ladenburg†, who describes the broadening of the principal line with the change of current intensity. We also noticed ourselves much change in the appearance of this line, when the condition of lighting of the Arons-Lummer lamp was changed from 30 volt 10 amp. to 100 volt 3 amp.

In the present work, the effect of diffraction and scattering due to the line as well as the slit was entirely neglected. This may not be negligibly small. The

* Gale and Lemon, Astrophys. Jour. **31**, p. 78, 1910.

† R. Ladenburg, Ann. d. Phys, **38**, p. 249, 1912.

absorption of light by the photographic plate and the film, as well as the sensibility of the film, were considered to be uniform throughout the length of the plate measured.

These may be considered as some of the defects of the method employed; but for a first approximation, the galvanometer indications may doubtless be taken as a measure of relative intensity.

An inspection of Fig. 4 will show at once such a remarkable proportionality of the intensity of some satellites to the distance from the principal line, that it seems to indicate some hidden physical connection between the two.

It would be interesting to compare our results with those obtained by visual observations, having recourse to some instrumental appliances such as Hartmann's microphotometer.

§ 9. *Satellites.* 5790:—Of the different lines examined in the present experiments, none presents such a complex structure as the line 5790. There are numerous lines in 5461 and 4359, but their distribution, as observed with the echelon only or crossed by the plate, is tolerably simple. With the yellow line 5790, however, the complexity is due to the existence of two satellites, both at about −1.0 Å. U. from the principal line. The stronger of the two has been noticed by several previous investigators, and both were directly photographed with a Michelson grating by Gale and Lemon.* In the course of our investigation, the stronger satellite was easily photographed by a Rowland concave grating (Radius 10 $\frac{1}{2}$ ft, 14438 lines to the inch), Fig. 5 showing its position with respect to the principal 5790 and the neighbouring line 5769, already in the first order spectrum. The distance of the satellite from the principal is about $\frac{1}{20}$ of the distance between the two strong lines. This line and its companion appear in the spectrum of the echelon grating mixed with other satellites (Fig. 6), which lie very near together and are of the same order as the principal, while the said satellite is two orders higher. To discriminate these lines from other faint lines, it is necessary to cross the echelon with the plate (Fig 7). By measuring the positions of the interference points by means of a micrometer, and joining the points representing different satellites by parabolas, which form loci of points for the same orders of spectra, we easily find that there are distinctly two series of points, which differ from other neighbouring points by belonging to different orders of spectra. The results of micrometric measurements are given in Fig. 8, and a copy of the original to the right (Fig. 7). The appearance of the echelon spectrum with the respective positions of other satellites is given below the diagram of the interference points in Fig. 8. The ghost is marked with the letter G. They are all blended together, so that it seems impossible to discriminate the distribution of the satellites with the echelon grating alone, as the diagram of the echelon spectrum will show.

Although the crossed spectra indicate the position of the said satellites

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* Gale and Lemon, Astrophys. Jour, **31**, p. 78, 1910.

beyond doubt, we tried to bring fresh evidence as regards their position in the echelon spectrum.

Instead of using the spectrum given by the Abbe prism in the echelon apparatus as constructed by Hilger, we have projected the line given by the concave grating, in which the stronger satellite -1.0 Å. U. was distinctly separated from the principal line, on the slit in front of the echelon grating, so that it can be either included or excluded in obtaining the echelon spectrum. The spectra with and without the said satellites are given in Fig.'s $9_a$ and $9_b$ respectively. They show that the third line from the left is due to the satellite about −1.0 Å. U distant. The fourth line is a little displaced in its relative position, which shows that the neighbouring satellite is almost coincident with the strong line. It may be worthy of remark that where there is some doubt as to the legitimacy of the position of a satellite, we may bring in the aid of an instrument of high dispersion and, by process of elimination, arrive at a correct result.

A discussion was raised by Gmelin[*] as to the existence of the satellite+0.164. By a method which is quite analogous to that of crossed spectra he showed that, in place of the said line, there is a satellite -0.374. With the echelon here used, and also with the crossed plate, the line falls in the neighbourhood of -0.938 and+0.224. Numerous photographs have been taken with different microplanars and photographic lenses, but it was difficult to see distinctly the exact position of the said satellite. There are several indirect evidences for its existence, but we have not given it in the figure, as its existence could not be directly ascertained.

As to the faint line+0.165, which appears distinctly in our crossed spectra, and which has been measured by Janicki[*], Galitzin[†], and Lunelund[‡], there is not the least doubt of its existence. It may be due to an accidental coincidence of the line with the alleged -0.374 in the echelon spectra of the above mentioned observers, that the line was cancelled by Gmelin. It is also very curious that -0.938, which first appears in the observations of Gale and Lemon, had not been noticed previously. In our case, it is altogether impossible to discriminate it from +0.224 in the echelon spectrum, but the interposition of the Lummer plate places its existence beyond doubt, as illustrated in the diagram of the interference points. The order of plate spectrum for 0.938 is much higher than that for the neighbouring point +0.224. This is of a special interest, showing, as it does, how crossed spectra can sometimes analyse the combination of several lines in a single echelon spectra into different interference points.

Instead of giving a table of the results of different observers, the coincidence, as well as the discrepancy, of different measurements are shown at a glance in the annexed figure. The thickness of the lines is drawn proportional to their intensity.

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* Gmelin, Ann. d. Phys. **33**, p. 17, 1910.

† Janicki, Ann. d. Phys. **19**, p. 36, 1906.

‡ Galitzin, Bull. d. l'Acad. Imp. d. Sc. d. St. Petersbourg, p. 159, 1907.

§ Lunelund, Ann. d. Phys. **34**, p, 505, 1911.

5790.

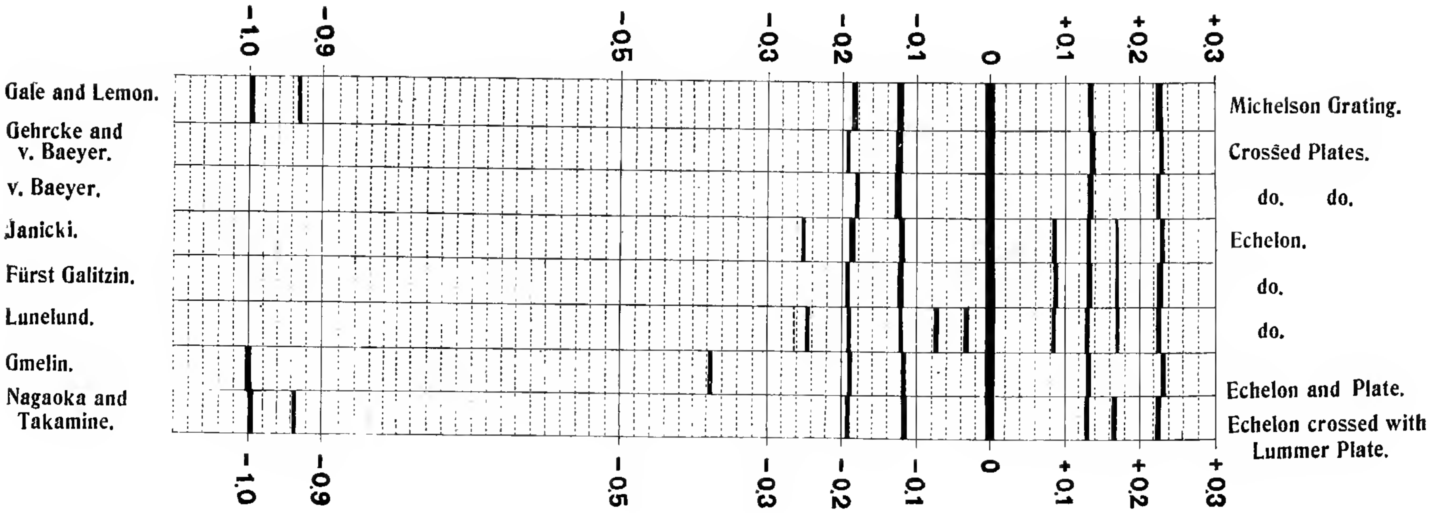



FIG. 10.

It is to be noticed that, the principal line being wide and diffuse, the mean point is difficult to determine; the consequence is that the distribution of the satellites by one observer is one-sided compared with that of another. The results of our own measurements are tabulated below:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Echelon | −0.996 | −0.931 | −0.196 | −0.129 | 0* | +0.132 | +0.161 | +0.217 |
| Plate | −0.999 | −0.938 | −0.191 | −0.119 | 0* | +0.130 | +0.164 | +0.224 |
| Intensity | 2 (?) | 1 (?) | 2 | 6 | 10 | 3 | 2 | 4 |

We consider that the results obtained by the plate are more accurate than those obtained by the echelon, inasmuch as the optical errors attending the former are simpler than those attending the latter and also as micrometric measurements can be made on numerous orders of spectra with the plate.

Gmelin showed that +0.084 of Janicki and +0.086 of Galitzin are coincident with the satellite -1.0 Å. U. Lunelund recognized that +0.082 of his observation would correspond to the above satellite, at the position

$$-2\, d\lambda_{max} + 0.082 = -1006 \text{ Å. U.};$$

but he had no means of discriminating its order from that of the other satellites.

It is questionable whether it is proper to treat this line as a satellite of 5790 or as a separate line. We are inclined to view it as the latter, and think it proper to consider the faint line −0.938 as a satellite of this line; *see* discussion of the values of $\frac{\delta\lambda}{\lambda}$, given below in § 10.

* The line marked by an asterisk is the reference line in the determination of $\delta\lambda$.

5769:—This line is quite simple; it has three satellites, of which two are nearly symmetrically placed about the principal line and also nearly equal in intensity. The third satellite seems as if it were the ghost of the principal, but micrometric measurement shows a distinct deviation from the position which it ought to occupy were it a false line due to the echelon.
Fig. 11 shows the crossed spectra and Fig. 12. the echelon spectrum.

Our measurements are as follows:—

| | | | | |
|---|---|---|---|---|
| Echelon | −0.109 | −0.049 | 0* | +0.046 |
| Plate | −0.121 | −0.050 | 0* | +0.044 |
| Intensity | 1 | 3 | 10 | 3 |

The lines +0.084 and +0.121 given by Lunelund did not appear in the crossed spectra, and are probably ghosts. The results of various observers are shown diagrammatically in Fig. 13.

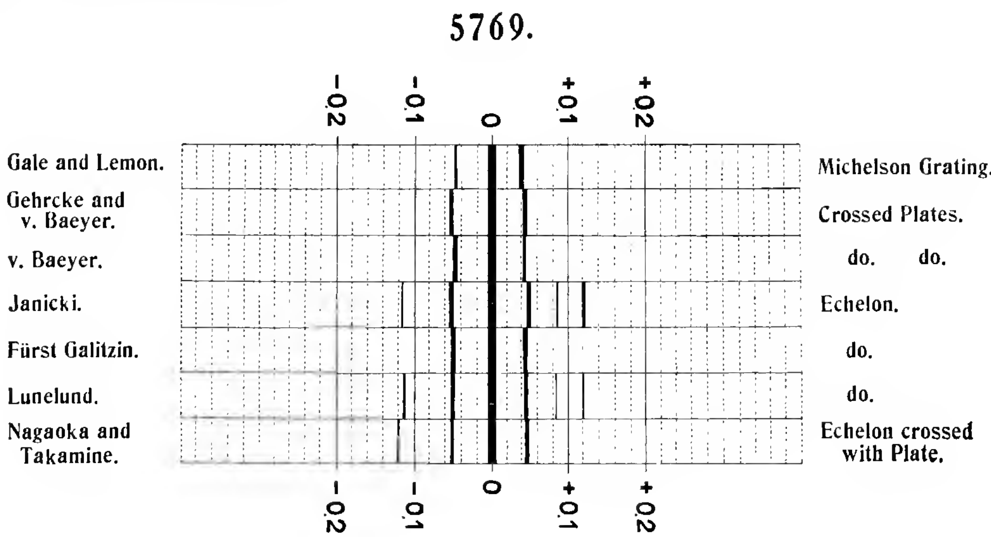



Fig. 13.

5461:—Much interest attaches to this line, as it is the brightest and the most accurately examined of the lines of mercury. The crossed spectra (Fig. 14), the diagram of the interference points (Fig. 15) and the echelon spectrum (Fig. 16) show at once the presence of eight satellites, the existence of which can hardly be doubted. In order to separate the principal line from its nearest companion −0.026, it is necessary to have the lamp at low voltage, and to place the line of sight in the direction of the arc. Also good cooling of the tube is necessary. The photographs of the echelon and of the crossed spectra taken under these conditions are shown in Fig.'s 17 and 18. By long exposure of the photographic plate, we were able to notice fine ghosts (indicated by G in the diagram annexed to Fig. 15), in the neighbourhood of the lines −0.242 and +0.210; these are very much like those observed by Stansfield,

who, however, considered the real line -0.054 also as a ghost. The advantage of crossing one spectrum with another is apparent in the examination of the faint lines as here stated. The faint line +0.210 is not easy to measure, but its existence is beyond question (Fig. 19).

The results of different observers are given in the form of a diagram in Fig. 20.

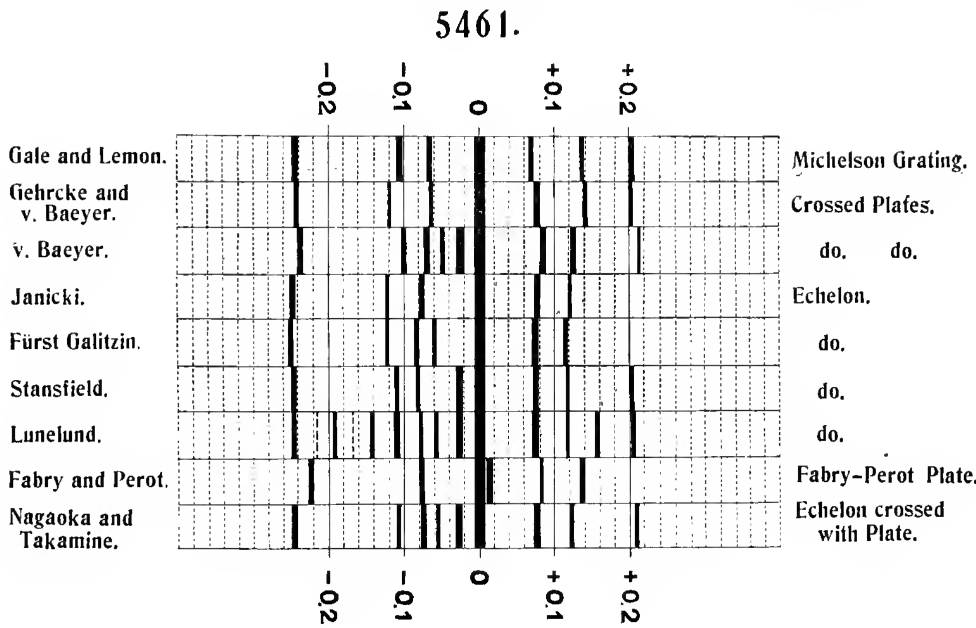



FIG. 20.

Our measurements are as follows :—

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Echelon | −0.246* | −0.113 | −0.080 | −0.055 | −0.026 | 0 | +0.072 | +0.118 | +0.204 |
| Plate | −0.242* | −0.108 | −0.074 | −0.054 | −0.026 | 0 | +0.078 | +0.123 | +0.210 |
| Intensity | 5.22 | 2.62 | 4.92 | 1.50 | 8.26 | 10 | 5.77 | 3.47 | 1.34 |

These numbers agree very well with those found by v. Baeyer. Several of the lines cited by Lunelund occupy positions coinciding with the ghosts in our echelon spectrum, and have probably no real existence.

Janicki* has recently discovered that the principal line of 5461 is quadruple. The separation of these finer lines was beyond the resolving power of our instruments, but we were convinced of the multiplicity of the line by crossing the Lummer-Gehrcke plate with a Fabry-Perot air plate of about 60 mm. thickness. Evidently the resolving power of the latter may be estimated at more than a million, while that of the former does not exceed 400000.

We shall afterwards learn that the intensities of different satellies are in regular order.

* Janicki, Ann. d. Phys. **39**, p. 439, 1912.

4359 :—This strong line is accompanied by ten satellites, so that the crossed spectra are by no means simple, but as they are not so widely apart from the principal as in 5790, their discrimination as to the order and position of the interference points is not so tedious as with the yellow line. The echelon spectrum is accompanied by five ghosts, (G in the diagram annexed to Fig. 24), which are quite intense enough to be easily mistaken for real lines, unless eliminated by crossing it with the Lummer plate. More than 100 photographs were taken before we were sure of the existence of distinct lines. Fig. 21 is a copy of the original, Fig. 22 the same, enlarged, whilst in Fig. 25 and Fig. 26 the higher orders of the crossed spectra are still more magnified. The photographs of the echelon spectrum (Fig. 23 and Fig. 27) indicate how misleading they are, if we have them only to rely on without some means of separating the false from the true lines. The accompanying diagram (Fig. 28), indicating the results of different observers on the distribution of lines in 4359, which distribution bears a close resemblance to that of the satellites of 5461, shows how unreliable the echelon spectrum alone is.

4359.

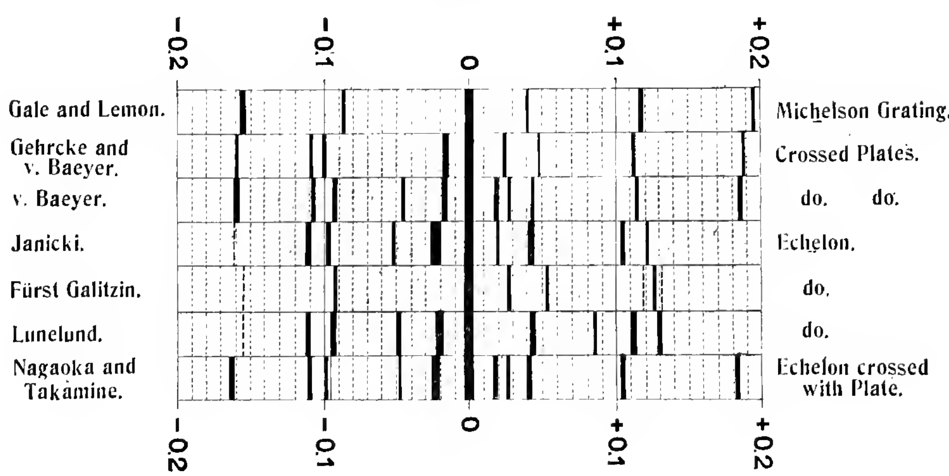



FIG. 28.

The following table gives our measurements.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Echelon | −0.161 | −0.108* | −0.094 | −0.045 | −0.019 | 0 | +0.020 | +0.032 | +0.045 | +0.106 | +0.183 |
| Plate | −0.163 | −0 110* | −0 097 | −0.097 | −0.023 | 0 | +0.026 | +0.017 | +0.043 | +0.105 | +0.182 |
| Intensity | 6 | 4 | 3 | 1 | 8 | 10 | 4 | 1 | 5 | 5 | 4 |

The number of lines, as well as their positions, agree tolerably well with the measurements of v. Baeyer, who had the command of the highest resolving power of all the experimenters above cited.

4078 :—The structure of this line is simple, as shown in the crossed spectra (Fig. 29), and in the echelon spectrum (Fig. 30).

The appearance of the latter figure reminds one of the similarity with 5790, if the line -1.0. Å. U. be eliminated. The discrepancies among different observers are not so large as in the case of other complex lines. Our measurements are given below.

| | | | | | | |
|---|---|---|---|---|---|---|
| Echelon | −0.079 | −0.048* | 0 | +0.029 | +0.044 | +0.067 |
| Plate | −0.077 | −0.047* | 0 | +0.032 | +0.050 | +0.076 |
| Intensity | 4 | 5 | 10 | 3 | 5 | 2 |

It is very singular that there is no discrepancy among different observers as to the existence of five satellites, although their positions differ slightly, as will be seen from Fig. 31.

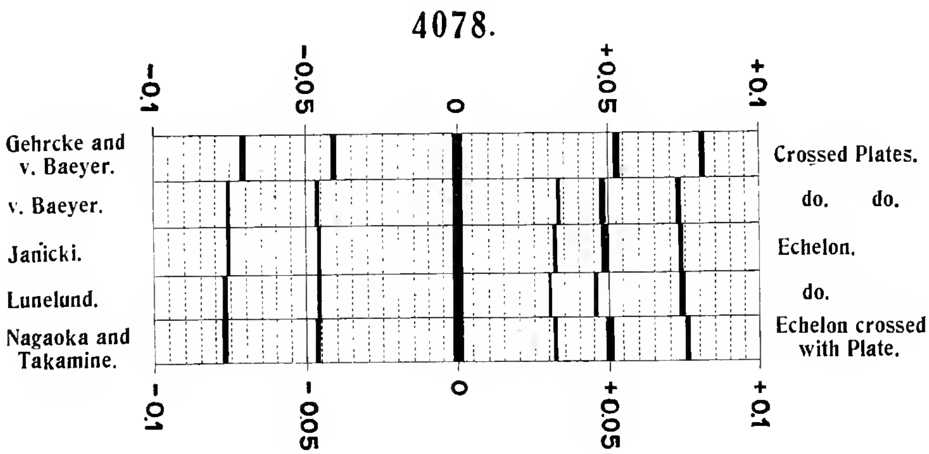



Fig. 31.

4047 :—The echelon spectrum of this line, by a long exposure, sometimes shows a number of ghosts. Fig. 33 gives its photograph taken under 15 min. exposure, and Fig. 32 the crossed spectra photographed with a lens of short focus. A lens of longer focus gives more detail. (Fig. 34). The principal line is found to be double, and near it is another doublet, which has been often considered as a single broad line. Recent experiments made by crossing the plate with the Fabry-Perot air plate place beyond doubt the duplicity of the principal line. The accompanying diagram shows the discrepancies among different observers in the position of the satellites. (Fig. 35.).

Although we have taken photographs under an exposure of several hours, we have not been able to establish the existence of the numerous satellites given by Wendt. It appears that they were mostly due to the nature of the lamp.

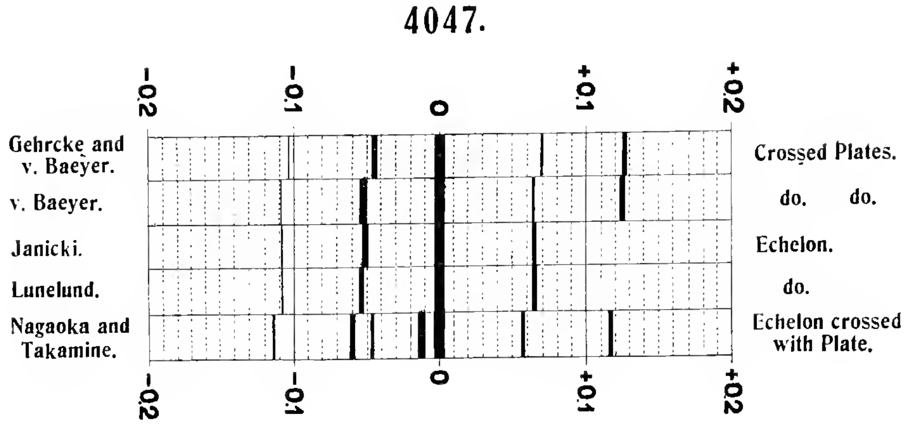



FIG. 35.

Our measurements are given below.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Echelon | −0.118* | −0.063 | −0.049 | −0.013 | 0 | +0.060 | +0.117 |
| Plate | −0.114* | −0.060 | −0.046 | −0.012 | 0 | +0.059 | +0.117 |
| Intensity | 3 | 7 | 4 | 8 | 10 | 5 | 5 |

We have further to remark that the line 4916 is single; 4348 and 4339 are difficult to photograph without the intrusion of the strong line 4359, so that it is not easy to determine how many satellites accompany them. Probably 4348 has three, but we tried in vain to get them separated from the lines of 4359, which inevitably appeared with 4348 as diffused light.

§ 10. *Regularity*. One* of us has already pointed out that the distribution of the satellites is not altogether irregular. This was however stated with reserve, as the ghosts were not eliminated and were misleading. In the present experiments, the ghosts have been eliminated, so that we may now enter into the discussion of the distribution of the satellites with confidence.

It has already been stated that the values of $\delta\lambda$ of some satellites are in a simple ratio; a good example is afforded by the satellites of the lines 5461, 4078 and 4047.

Distinguishing the satellites by numbers, we find that $\delta\lambda$'s can be expressed as multiples, or sometimes as combinations of $\delta\lambda$ of other satellites, as given in the fourth column of the table below :

* Nagaoka, Phys. Zeit, **10**, p. 609, 1909.

| | Satellite | δλ (observed) | δλ |
|---|---|---|---|
| Line. 5461...... | (−5) | −243.2 | =−(+2)×2+2.6 |
| | (−4) | −108.3 | = (−1)×4+2.3 |
| | (−3) | − 75.0 | =−(+1) +2.0 |
| | (−2) | − 55.8 | = (−1)×2−2.8 |
| | (−1) | − 26.5 | |
| | (+1) | + 77.0 | =−(−1)×3−1.5 |
| | (+2) | +122.4 | |
| | (+3) | +209.6 | =−(−4)×2+7.0 |

$\delta\lambda$'s are expressed in m. Å. U.

We obtain the following approximate relation:—

(−1): (−2): (−3) or (+1): (−4)=26.5: 55.8: 75.0 (or 77.0): 108.3=1: 2: 3: 4

The ratio is not exact, but it does not seem to be a mere chance-coincidence.

| | Satellite | δλ (observed) | δλ | |
|---|---|---|---|---|
| Line. 4078...... | (−2) | −76.9 | =−(+3) | −0.7 |
| | (−1) | −46.9 | =−(+2) | +3.1 |
| | (+1) | +32.0 | =−(−2)+(−1)+2.0 | |
| | (+2) | +50.0 | =−(−1) | +3.1 |
| | (+3) | +76.2 | =−(−2) | −0.7 |

Lines (−2) and (+3) are symmetrical about the principal; (−1) and (+2) are nearly so; (+1) is given by the difference between (−2) and (−1).

| | Satellite | δλ (observed) | δλ | |
|---|---|---|---|---|
| Line. 4047 ... | (−4) | −114.1 | =−(+2) | +2.6 |
| | (−3) | − 60.1 | =−(+1) | −1.6 |
| | (−2) | − 46.4 | =−(+1)−(−1)−0.3 | |
| | (−1) | − 12.4 | =−(×1)−(−2)−0.3 | |
| | (+1) | + 58.5 | =−(−3) | −1.6 |
| | (+2) | +116.7 | =−(−4) | +2.6 |

The approximate symmetry between (−4) and (+2), (−3) and (+1) is at once evident, and (+2)=2×(+1).

If we investigate the structure of other mercury lines, we may perhaps find a similar relation; we believe, further, that such relation is not only peculiar to mercury lines, but is also to be found in the lines of other elements; in fact, manganese affords such an instance, as investigated by Janicki. Examples of the sym-

metrical positions of the satellites with respect to the principal line are to be seen in Fig.'s 10, 13, 20, 28, 31, and 35. This symmetry of position is however not always attended with equality of intensity.

In addition to this, the character of the distribution of the lines is similar, especially in 5461 and 4359, which belong to the second subordinate series. In both of these lines, the principal line is accompanied by a strong satellite on the side towards the violet, and very close to it. For these two satellites, the difference in frequency from their respective principal lines is nearly the same, which is also characteristic of some alkaline elements.

Thus for 5461 : $\frac{\delta\lambda}{\lambda^2}=88\times10^{-11}$,

4359 : ,, $=89$ ,,

4047 : ,, $=76$ ,,

The last line 4047 shows some deviation, but as the position of the neighbouring satellite is so near that it is hardly resolved by the instruments at our command, we must wait for a more accurate determination with instruments of a greater resolving power.

Another characteristic is that the values of $\frac{\delta\lambda}{\lambda}$ are common to several of the satellites of different lines, and are multiples of those preceding them; so that some numbers occur oftener than others, in a table of $\frac{\delta\lambda}{\lambda}$ constructed for different lines.

| $\lambda$ | $\frac{\delta\lambda}{\lambda}\times10^7$ |
|---|---|
| 4047 | 31 |
| 4359 | 40 |
| 5461<br>4359 | −49<br>−53 |
| 4359 | 60 |
| 5769<br>4078<br>4359 | 77<br>78<br>2×40=80 |
| 5769 | −87 |
| 5461<br>4359<br>5461<br>4359 | −102<br>98<br>−2×49=−98<br>−2×53=−106 |
| 4359<br>4348 | −110<br>−110 |
| 4348<br>4078<br>4047 | 115<br>−115<br>−115 |

| $\lambda$ | $\frac{\delta\lambda}{\lambda}\times10^7$ |
|---|---|
| 4078<br>4359<br>4359<br>4047 | 122<br>2×60=120<br>3×40=120<br>4×31=124 |
| 5461<br>5461<br>5461<br>4047<br>4047 | −138<br>141<br>−3×49=−147<br>145<br>−149 |
| 4348<br>4078<br>4078 | 186<br>187<br>−188 |
| 5461<br>5461<br>5461<br>4359 | −198<br>4×49=196<br>−2×102=−204<br>2×98=196 |
| 5790<br>5769 | −206<br>−210 |
| 5790<br>5461<br>4359<br>4359<br>4348 | 223<br>224<br>−222<br>−2×110=−220<br>−2×110=−220 |

| $\lambda$ | $\frac{\delta\lambda}{\lambda}\times10^7$ |
|---|---|
| 4359<br>4359 | 240<br>4×60=240 |
| 4359 | =253 |
| 5790<br>4047<br>4047<br>4047 | 283<br>−282<br>289<br>2×145=290 |
| 5790<br>4359<br>4358 | −330<br>3×110=330<br>3×110=330 |
| 4359<br>5461<br>5790<br>4348<br>4078 | −374<br>383<br>386<br>2×186=372<br>2×187=374 |
| 4359<br>5790<br>5769<br>5461 | 416<br>−2×206=−412<br>−2×210=−420<br>−3×138=−414 |
| 5461<br>5461<br>4359<br>4359<br>4348 | 414<br>2×224=448<br>−2×222=−444<br>−4×110=−440<br>−4×110=−440 |
| 5790 | −1620 (?) |
| 5790 | −1725 (?) |

The values of $\frac{\delta\lambda}{\lambda}$ in the last two lines of the above table refer to $-0.999$ and $-0.938$ of 5790, and shew that they do not fit in the numbers common to the other lines. It seems reasonable to regard $-0.999$ as the principal line, and $-0.938$ as its satellite, with $\delta\lambda = +0.061$. Then $\frac{\delta\lambda}{\lambda} \times 10^7 = 105$, which is also observed in the lines 5461 and 4359, as given in the table.

The above table exhausts all the values of $\frac{\delta\lambda}{\lambda}$ for the satellites found in the present experiments. For 4348, the mean of the values obtained by different observers was taken.

It will be seen that some of these numbers are both remarkably coincident for several lines and multiples of the others.

Interpreted in the light of Doppler's principle, the quantity $\pm\frac{\delta\lambda . c}{2\lambda}$ gives the velocity $u$ of the approach or recession, $c$ denoting the velocity of light. The numbers above obtained seem to show that $u$ range from several hundred metres to many thousand metres per second. But this is only a suggestion, and we do not mean to affirm that the above is really due to the Doppler effect. The multiplicity of the above numbers may probably be due to the multiplicity of electronic charges, which form the centers of light vibration. It would, however, be premature to speculate upon any theory in explanation of the facts here described.

It may be suggested that the change of frequency, which is proportional to $\frac{\delta\lambda}{\lambda^2}$, will have the same property as $\frac{\delta\lambda}{\lambda^2}$; a table of $\frac{\delta\lambda}{\lambda^2}$ was constructed, but as yet no inference could have been drawn from it, except the one already mentioned with respect to the companion of the principal line.

The intensities of many satellites seem to follow a simple law. A glance at Fig. 4, representing the intensities of the green lines referred to the principal line, will show that they decrease proportionally to $\delta\lambda$ from the principal line for the satellites $-0.026$, $-0.074$, and $-0.108$ on the negative side, and for $+0.078$ and $+0.123$ on the positive side. Again, the vertices of the intensity curves for $-0.242$, $-0.108$, and $-0.054$ lie on a straight line, which makes us doubt if these lines are not the satellites of the strong line $-0.242$. Only the satellite $+0.210$ does not fit in with these lines. (Fig. 36).

It must not, however, be forgotten that the ghosts in an echelon spectrum disturb the intensity of the neighbouring line to some extent.

The law of the proportionality of intensity to $\delta\lambda$ is also indicated in the intensity diagram for 4359, 4078, and 4047; owing to the complex structure of 5790, it is difficult to draw any inference for it and also for 5769, in which the satellites are too few in number.

With the line 4359, the intensity curve for $-0.023$ and $-0.097$ lies on a line with the principal, and also for $-0.163$, $-0.110$ and $-0.048$. Perhaps the latter lines are the satellites of $-0.163$. On the positive side, $+0.017$ and $+0.026$ lie on a line with the principal.

With the line 4078, $+0.050$ and $+0.076$ form one line with the principal, while the last line may also be grouped with $-0.047$ and $+0.032$. With 4047, only $-0.012$ and $-0.046$ form a group with the principal line.

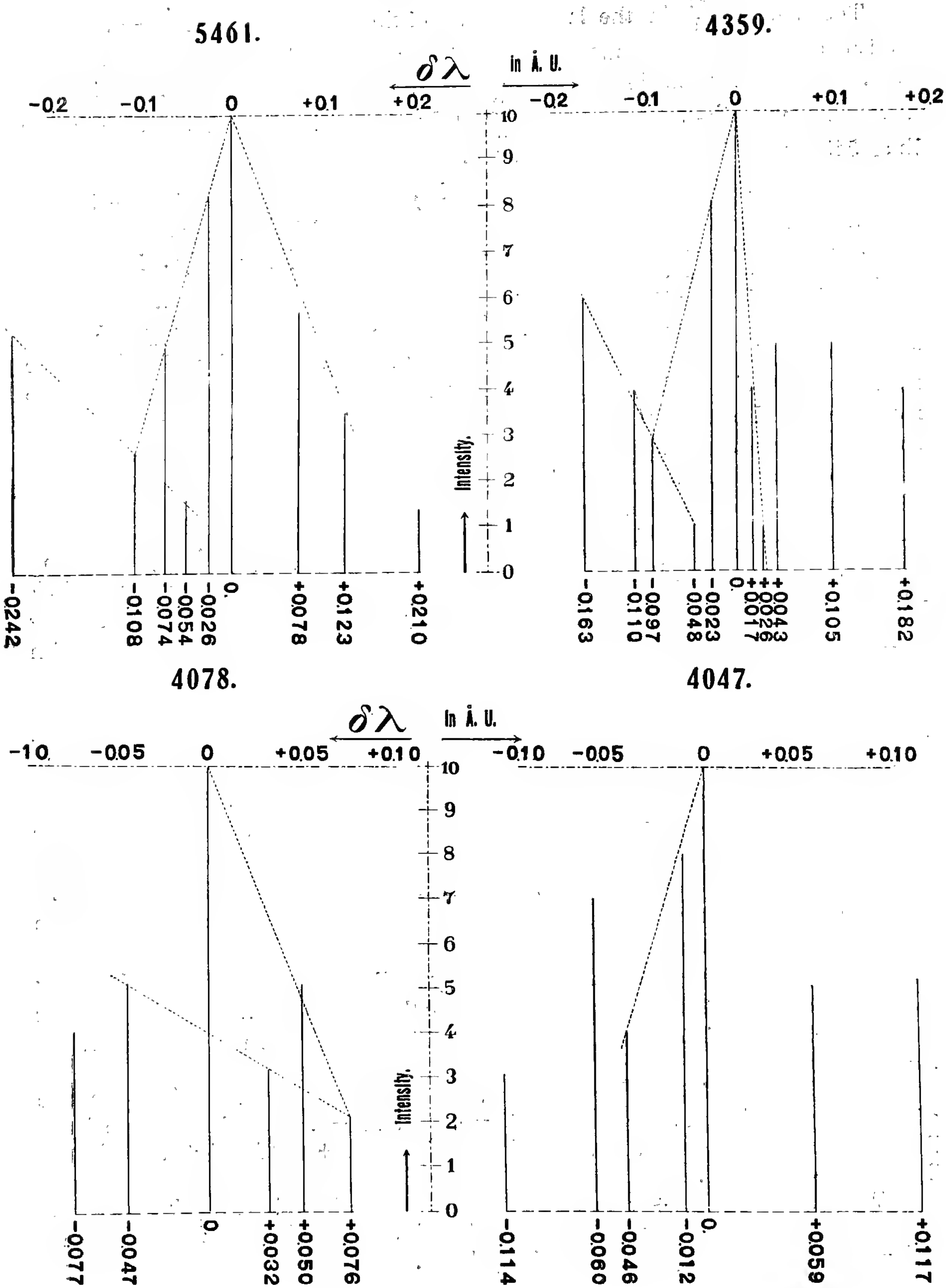

5461.
4359.
δλ
in Å. U.
-0.2
-0.1
0
+0.1
+0.2
Intensity.
-0.242
-0.108
-0.074
-0.054
-0.026
0.
+0.078
+0.123
+0.210
-0.163
-0.110
-0.097
-0.048
-0.023
0.
+0.017
+0.026
+0.043
+0.105
+0.182
4078.
4047.
-0.10
-0.05
+0.05
+0.10
-0.077
-0.047
+0.032
+0.050
+0.076
-0.114
-0.060
-0.046
-0.012
+0.059
+0.117

FIG. 36.

We have further to remark that in all of these lines, two satellites of almost equal intensity, which is nearly a half of that of the corresponding principal line, are always to be found; e. g., $-0.242$ and $-0.074$ in 5461, $+0.043$ and $+0.105$ in 4359; $-0.047$ and $+0.050$ in 4078, and $+0.059$ and $+0.117$ in 4047.

So far as we are aware, these relations as regards the positions and the intensities of the satellites, are now published for the first time.

It would be interesting to measure the intensities of the satellites with different apparatus, as the results here obtained may have been affected by instrumental errors.

At the present stage, we do not wish to advance any hypothesis regarding the distribution of intensities among the satellites, but we do wish to suggest that this distribution will have an important bearing on the explanation of the structure of complex spectrum lines, especially from the standpoint of the theory of ionic collisions.

It would also be interesting to see if these relations hold also for the satellites of the different lines of metals, such as bismuth, cadmium, lead, and others.

§ 11. *Conclusion.* From the results of previous observers and from the present experiments, we see that the echelon spectrum is almost invariably accompanied by ghosts and is ambiguous as regards the order of spectrum, when the satellites are not very near the principal.

These defects may be avoided by adopting the method initiated by Gehrcke and v. Baeyer, in which the echelon is crossed with the Lummer-Gehrcke plate, and its high resolving power as well as the brightness of the spectrum obtained by it is utilised.

The positions and intensities of the satellites of mercury lines are not entirely irregular; they seem to have some definite structure if properly arranged.

It is extremely desirable that similar researches should be extended to the spectrum lines of other heavy metals and that the conditions, under which the satellites appear or disappear, should be fully investigated.

The echelon spectroscope used in the present experiments was placed at our disposal by the Imperial Academy, to which the instrument belongs, and the Lummer-Gehrcke plate was procured from the memorial fund of the late Dr. Rokushiro Tsuruta. The experiments were made at the Physical Institute of the Imperial University, Tokyo.

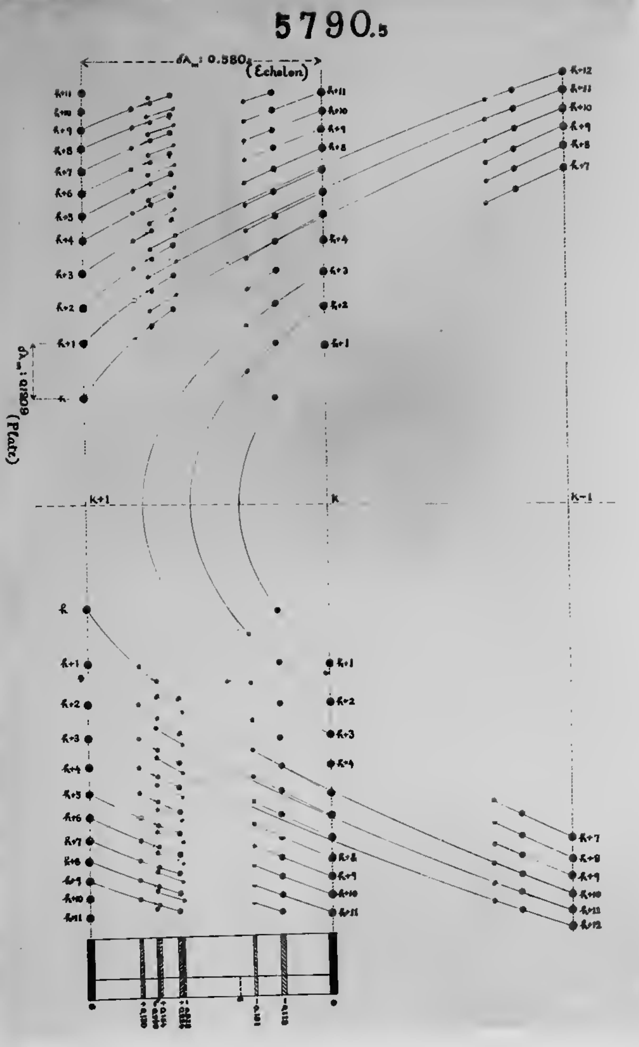



Fig. 8.

Fig. 7. Mag. 10.

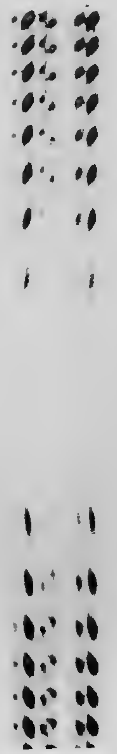


Fig. 11. Mag. 12.

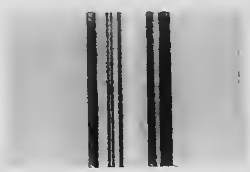


Fig. 6. Mag. 7.

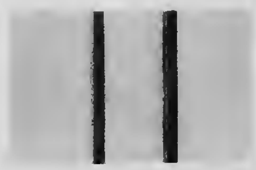


Fig. 12. Mag. 7.

Fig. 5. Mag. 8.

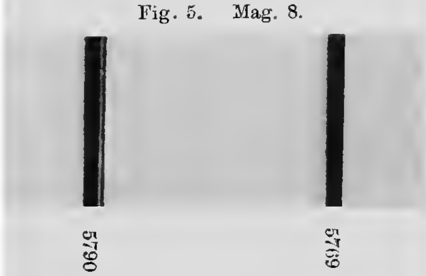



Fig. 9 *a*. Mag. 12.

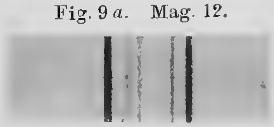
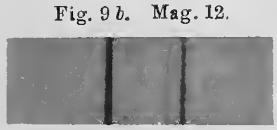


Fig. 9 *b*. Mag. 12.



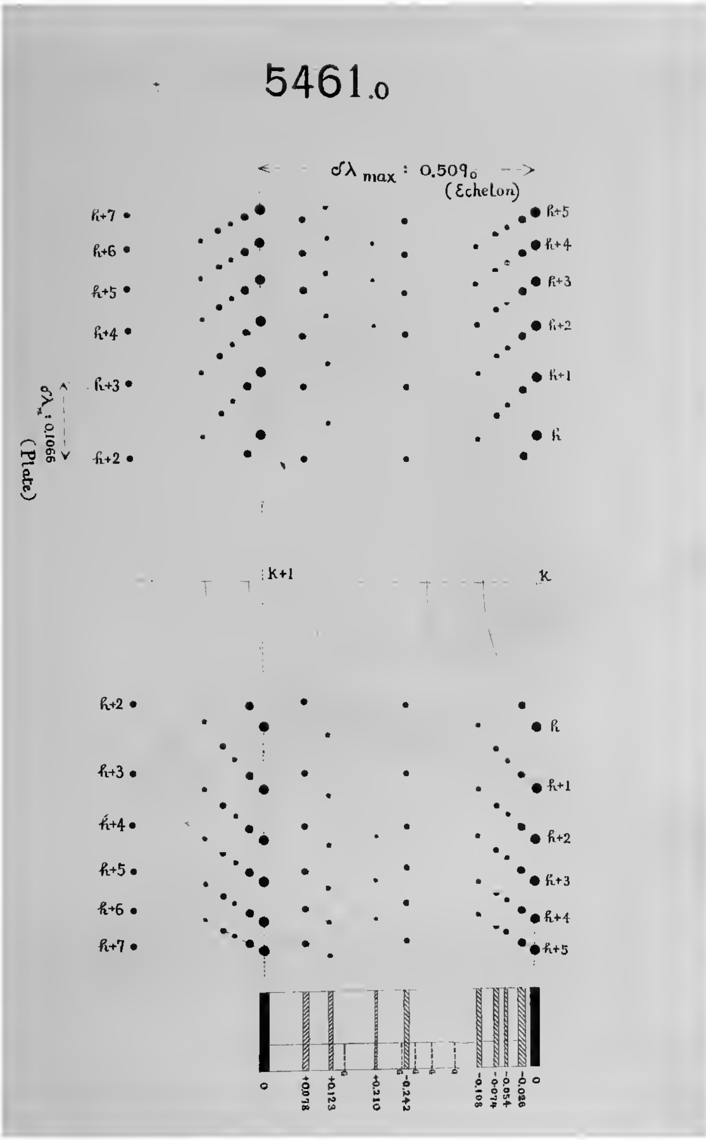



Fig. 15.

Fig. 14. Mag. 12.

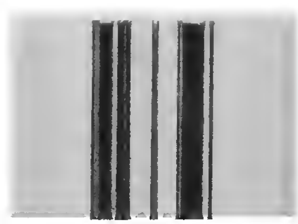


Fig. 16 *a*. Mag. 8.

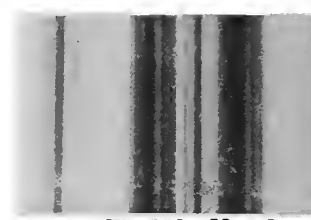


Fig. 16 *b*. Mag. 8.

Fig. 19. Mag. 10.

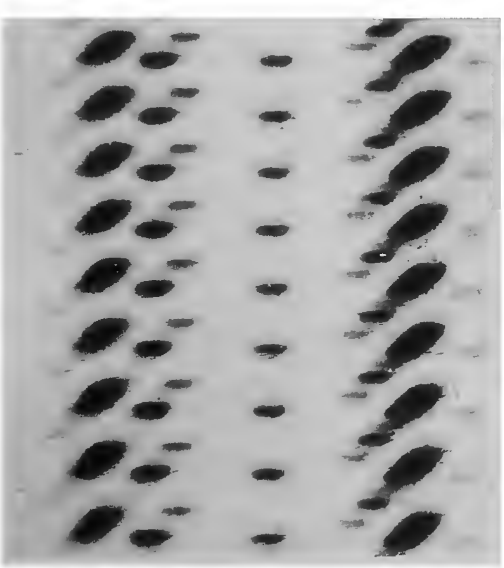
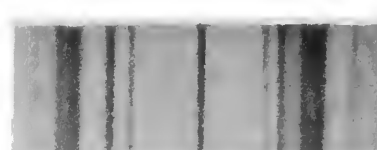


Fig. 18. Mag. 8.

Fig. 17. Mag. 8.



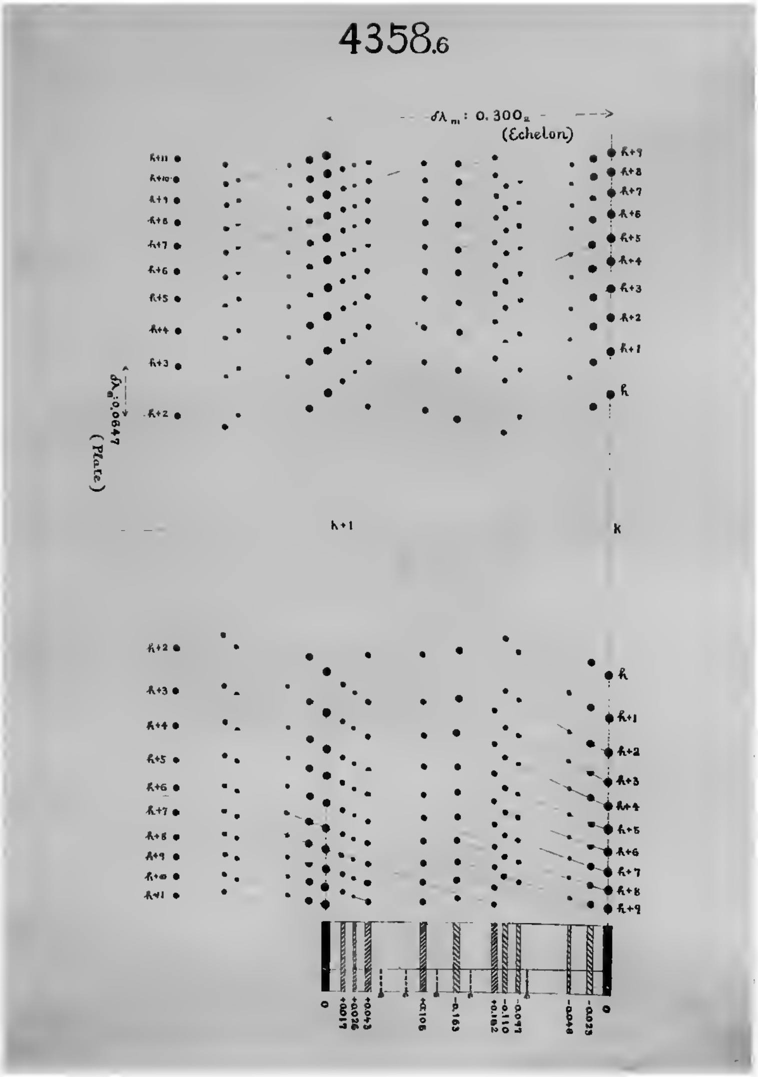
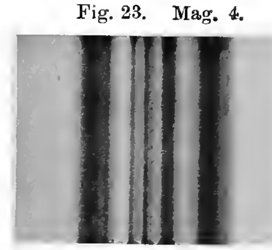



Fig. 24.

Fig. 22. Mag. 10.

Fig. 21. Actual Size.

Fig. 23. Mag. 4.



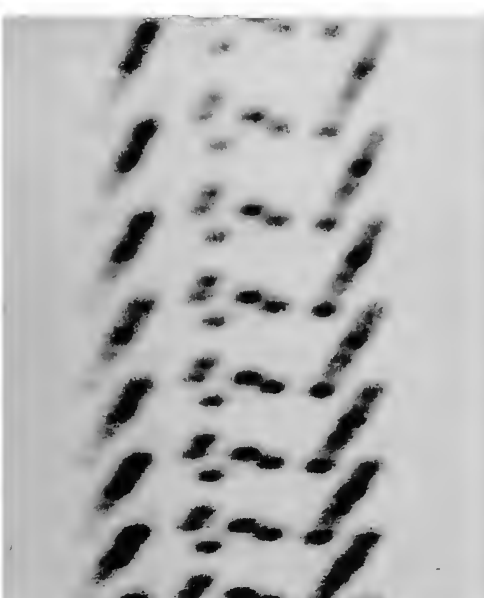


Fig. 25. Mag. 8.

Fig. 26. Mag. 8.

Fig. 27. Mag. 4.

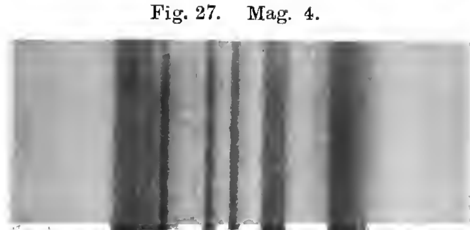


Fig. 34 *b*.

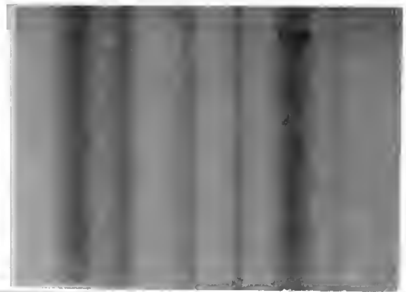


Fig. 34 *a*.
Mag. 8.

Fig. 33. Mag. 10.

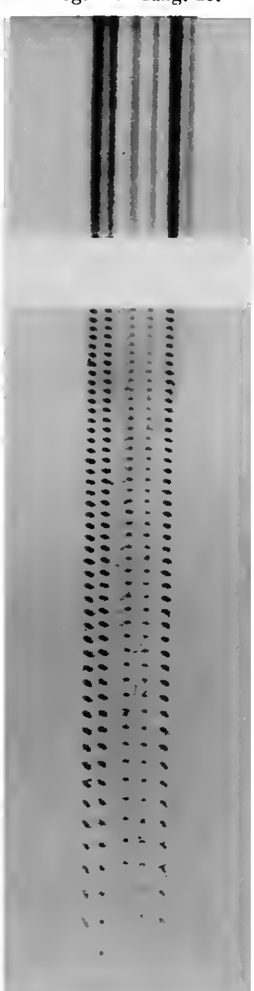
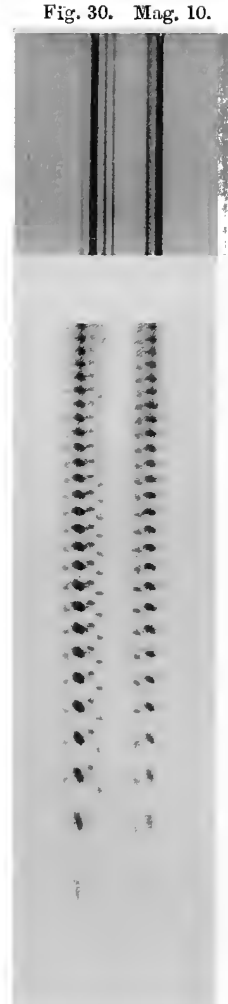


Fig. 30. Mag. 10.



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